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ABSTRACT

Schumacher and Nash (1991) theorize that writing provides an avenue for people to reorganize their knowledge. Kieren (1988) is one of several researchers who theorize that children's school fraction experiences should begin with personal knowledge of fractions. The purpose of this study was to determine whether or not children who engaged in journal writing about fractional situations showed an increase in fraction knowledge. Two groups of fourth-grade children at a large, Southwestern United States, urban elementary school participated in the study. One group completed a four-week intermittent writing experiment. The other group had no such experience. The results of this study indicated a significant difference between pre- and post-test scores for the writing group children. The results add evidence to the theory that writing causes learning and call teachers to provide writing situations associated with children's existing informal knowledge of fractions. (Contains 35 references.) (Author/ASK)

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RESULTS OF USING WRITING TASKS TO ENHANCE FOURTH-GRADE CHILDREN'S ACQUISITION OF FRACTION KNOWLEDGE

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Schumacher and Nash, (1991) theorize that writing provides an avenue for people to reorganize their knowledge. Kieren (1988) is one of several researchers who theorize that children's school fraction experiences should begin with personal knowledge of fractions. The purpose of this study was to determine whether or not children who engaged in journal writing about fractional situations showed an increase in fraction knowledge. Two groups of fourth-grade children at a large, southwestern United States, urban elementary school participated in the study. One group completed a four-week intermittent writing experiment. The other group had no such experience. The results of this study indicated a significant difference between pre- and post-test scores for the writing group children. The results add evidence to the theory that writing causes learning and calls teachers to provide writing situations associated with children's existing informal knowledge of fractions.

Introduction

Communication includes reading, listening, writing, and speaking about a topic. Mathematical writing is one type of communication that is encouraged by several mathematics education reform movements (e.g., National Council of Teachers of Mathematics, Mathematical Association of America). The purpose of this paper is to discuss the results of using writing as a method to facilitate the learning of fractions by fourth-grade children.

Theoretical Background and Literature Review

Writing

Schumacher and Nash (1991) posit *Construction of Multiple Representations* as a theory about students' restructuring of knowledge in a new and meaningful way through the writing process. In their theory, "writers in the process of composing create and transform multiple representations of meanings" (p.80-81). The Flower and Hayes (1986) cognitive model of writing consists of three major components: (1) task environment, (2) long term memory, and (3) working memory. Through interactions of these three components of the writing process, writers construct the explanations for their ideas, edit their words, and hence restructure their knowledge of the topic.

Knowledge of these ideas about writing can guide a teacher in development of writing assignments. The teacher should decide between two general objectives for engaging students in writing tasks: (a) writing as a medium *to communicate* known information about a topic and (b) writing as a process *to learn* new information about a topic (Applebee, 1984: Durst & Newell, 1989). According to *Construction of Multiple Representations* theory, learning occurs through any writing. But the nature of the learning will depend on the teacher's primary motivation for creating the writing task environment. The learning may manifest itself as greater facility with communication about the information or greater knowledge and understanding of the information itself.

Fractions

One of the primary pre-requisite concepts for learning about fractions is a sense of sharing (Hunting & Sharpley, 1988; Pothier, & Sawada, 1983). Children arrive at school with a sense of sharing. This early understanding of sharing is usually based on some manifestation of whole numbers (Behr, Harel, Post, & Lesh, 1992; Kieren, 1988). Therefore, teachers must take this knowledge into account while planning instruction that would move children toward acquisition of rational number concepts. According to Pothier and Sawada (1983), at least four levels exist in a child's mental growth in this area. *Sharing* or *partitioning* sets the stage for learning about fractions. *Halving* is a slightly more efficient skill that enables children to move to evenly partitioning a region into any power-of-2 pieces. The skill and concern over the need for same sizes of regions for even sizes other than powers-of-two appears at level 3 (*Evenness*). Finally children begin to develop strategies for equally and exhaustively partitioning regions into any number of pieces (*Oddness*).

A teacher might use this developmental theory to plan students' classroom experiences for fractions. However, the teacher also needs to consider the informal fractional knowledge a child might possess based on his or her "real life circumstantial knowledge" (Mack, 1995, p. 422) prior to selecting specific situations for the lessons. Moreover, learning experiences must be built upon individual students' unique fractional knowledge (Kieren, 1988; Streefland, 1978) and informal knowledge (Mack, 1990). Otherwise, previously learned rote procedures can interfere with children's successful acquisition of fraction concepts in problem solving experiences (Mack, 1990). Even when concrete and pictorial representations are specifically designed to facilitate the fractional information, the teacher must continually monitor students' fractional thinking and language (Hiebert, Wearne, & Taber, 1991) and the teacher must facilitate students' appropriate use, alteration, or creation of the visual aid in resolving fractional situations (Kamii & Clark, 1995).

Writing and mathematics

Writing is an appropriate mathematics activity because it enhances the writer's skills in communicating mathematics (NCTM, 1989) and it can involve all students (Connolly, 1989). Many researchers have studied the impact of writing in mathematics class. McIntosh (1991) and Rowan, Mumme, and Shepherd (1990) studied students who had completed mathematical writing as a means to demonstrate communication skill. In their study, students were directed to describe and explain mathematics and mathematical procedures used in solving problems.

Writing to learn activities have been successfully used with elementary children (Evans, 1984) as well as calculus students (Mett, 1989). So, as learners successfully engage in the writing process, mental structuring or re-structuring of the mathematics occurs and the learner comes to "own" the topic or idea.

In these cited studies, researchers performed little analysis of these students' mathematical writings to determine any correspondence to the mathematical knowledge gained. None of the cited studies about fractions are based on writing. The purpose of this study was to determine the sole effectiveness of fourth grade children's journal writing about fractions, exclusive of regular instruction, with regard to the children's acquisition of formal fractional knowledge.

Methodology

Sample

Two in tact groups of fourth-grade children at a large, southwestern United States, urban elementary school participated in this study. The first group, Group A, which served as the writing group, had 20 fourth grade children enrolled. The purpose of the study was to consider the impact of writing about mathematics on the students' mathematical understanding. So, only data generated by students who had previously demonstrated proficiency in writing were included in the analyses. Fourteen children completed all writing tasks and had previously passed the written composition portion of the Texas Assessment of Academic Skills.

The second group, Group B, which served as the comparison group, was also composed of 20 students. In order to appropriately compare the two groups, data analyses included only data generated by the 12 children who had previously passed the written composition portion of the Texas Assessment of Academic Skills.

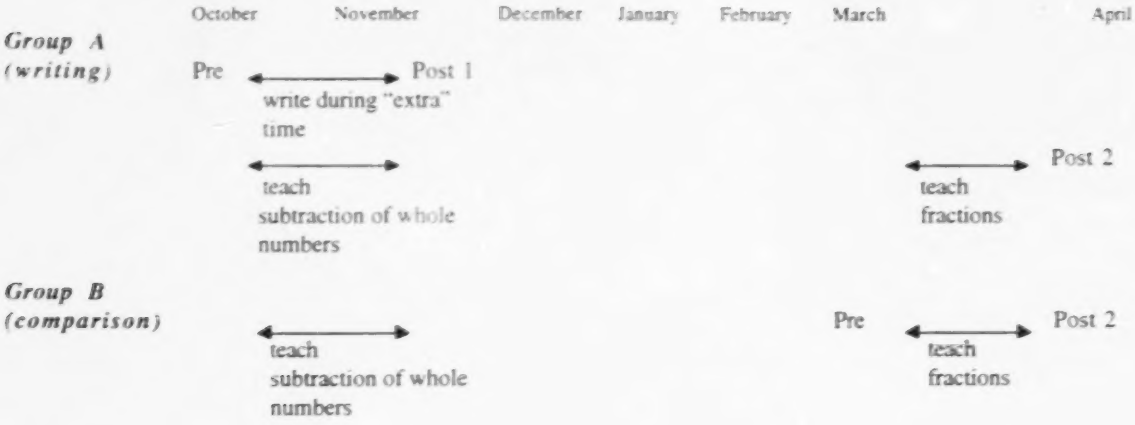
The classes were taught by different teachers with similar educational backgrounds: a language arts teaching emphasis and over five years of teaching experience. Both teachers were considered by their peers to have excellent rapport with students.

Design

This exploratory, field-based study was based on a two-group, repeated measures, design. Because this study was to be completed in a school environment, in a classroom already established, it came complete with all of the design problems associated with in school studies and attempts at experimental designs. Group A received the educational intervention described below. Group B was recruited to address some of the validity concerns association

with this action research study in which no random assignment could be completed. Figure 1 provides a visual time-line for the procedures of this study.

Figure 1 - Timeline of Study



Procedures

The writing experiences were conducted prior to students' formal fourth grade encounters with fractions. For group A, the writing experience with fractions did not replace or coincide with classroom instruction in whole number subtraction. The actual writing was completed during a school-wide allotted "extra practice" time. This time is typically used at the discretion of each classroom teacher. Group B's "extra practice" time was dedicated to academic activity such as completion of homework.

The day before the first day of the writing experiment, the students in Group A took the pre-test (Pre) over fractions. The intermittent writing took place during a four week period, with students responding in their journals to nine fraction experiences/prompts. The long time frame was included in the design in order to concede Carraher's (1996) caution, "Concepts as rich as fractions are not acquired immediately, because they require reorganization of prior knowledge" (p. 264). The day after the final writing assignment, the students took the post-test, (Post 1).

Five months later, both the comparison group (Group B) and the students who had experienced the journal writing (Group A) encountered fractions through teacher-led instruction. Prior to the first day of the three weeks set aside for regular instruction, the comparison group took the same pre-test previously taken by the writing group. After three weeks of regular instruction, both groups took the post-test (Post 2) over fractions.

Materials

Acquisition of fraction knowledge was measured by the three different tests (Pre, Post 1, and Post 2). Two of these examinations (Pre and Post 1) were prepared by the textbook authors (Eicholz, O'Daffer, Charles, Young, Barnett, Fleenor, Clemens, Thornton, Reeves, & Westley, 1991). The third examination (Post 2) was created, problem by problem, based on the textbook examinations. All three tests contained 12-multiple choice items. All tests were judged by the two teachers and the researcher to be equivalent to, but different from, each other.

The teacher for Group A and the researcher jointly developed the series of in-context writing situations related to each of nine lessons from the thirteen-lesson chapter. Partially drawing from Streefland's (1982) work, the created writing prompts were intended to display the following characteristics:

- focus the goal but encourage lengthy explanations
- encourage describing situations in fractional language
- respect the informal knowledge students held
- follow the textbook chapter's sequence of topics
- use both real-world and synthetic situations

The related writing prompts, shown in Table 1 were based on those objectives.

Table 1 - Writing Prompts

Day 1 • Show students a clear plastic bottle that is about 1/2 full. Ask them to describe the bottle. Fill the bottle to the top. Ask them to describe the new situation • Take a square sheet of paper. Fold it in half, then cross-fold in half and cut 1/4 off. Ask them to describe the paper. • Ask them to read their paragraph and look for mathematical and fractional descriptions. • Ask students to define numerator, fraction (region definition), denominator, & estimating fractions in their own words.
Day 2 • Show students a box of crayons (count = 16). Place 10 crayons inside the box and leave 6 of them outside the box. Ask them to describe the situation. • Tell students to identify the fraction in the box and outside of the box. • Ask students to define fraction (set definition) & set in their own words. • Show students a set of paints (count = 12). Remove 4 of the colors and leave 8 colors in the box. Ask them to describe the situation.
Day 3 • Place a candy-bar in plain view of all students. Break it into two parts, which are clearly not equal and tell the student a sibling gets the bigger piece. Ask the students to write a letter to their parent about this situation. • Instruct students to write a postscript indicating how they would rectify the situation

Table 1 continued

Day 4 • Show students a piece of paper folded in half and colored on one of those halves. Then, fold again (to get fourths). Ask them to do the same thing and to describe the amount of the colored part.
Day 5 • Show students the procedure for finding equivalent fractions with 3/6 = 9/18. Ask them to explain the procedure for finding the denominator of 18.
Day 6 • Use patterns blocks to show students two blue parallelograms (2/3 of the whole hexagon) set on top of a yellow hexagon and four green triangles (4/6 of the whole hexagon) set on top of another yellow hexagon. Ask students to explain why these fractions are equal. • Teacher provides procedure for finding lowest terms. Ask students to use the above situation to explain lowest terms.
Day 7 • Give each student a sheet of graph paper and direct students to draw eleven 1 x 12 rectangles. Color representations of 1/12, 2/12, 3/12, ... 11/12. Ask students to describe two ways for finding lowest term representations of the fraction of their choice.
Day 8 • Show students twelve red counters in a 2 x 6 rectangular array. Demonstrate 2/6 by replacing the first two counters in each row with a white counter. Ask students to write a "how to" paragraph for finding 2/6 of 12.
Day 9 • Put 1 circle (yellow), 2 halves-of-a-circle (tan) and 1 third-of-a-circle (green) on the board. Ask them to describe how much is there altogether.

Early prompts were aimed at Pothier and Sawada's lower levels of halfing (water bottle), sharing (candy bar), and evenness (crayons & paper folding) with later prompts emphasizing oddness (fraction circles and equivalent fractions). An initial assumption of the researchers was that many of these students would have had little or no experience writing about mathematics. According to the Flower and Hayes model of writing, writers need a clear goal, topic, audience, and motivation. Consequently, the early multi-part writing prompts directed the students to write definitions in their own words or to look back over their descriptions and to search for the fractions in those descriptions.

Because these fourth-grade students had most previous experience with descriptive writing, 7 of the writing prompts elicited simple descriptions. Two later writing prompts were based on "how-to" paragraphs and personal letters.

Results

The formal fractional knowledge of both groups of children were evaluated according to changes in and comparisons of the students' pre- and post-test scores on the chapter examinations. The mean test scores and standard deviations are shown in Table 2.

Table 2 Mean and Standard Deviation of Test Scores

	Pre-test	**Post-test 1**	**Post-test 2**
Writing Group			
(n = 14) *Mean*	6.79	7.64	8.86
Standard Deviation	2.97	1.15	1.94
Comparison Group			
(n = 12) *Mean*	7.67		8.67
Standard Deviation	1.67		1.92

Due to small sample sizes (n = 14 and n = 12) and lack of random assignment, the three sets of within-groups pre-post test data were analyzed via a two-tail ($\alpha = .05$) correlated t-distribution. The correlated t-test revealed two significant positive correlations. The relationship between the writing group's pre-test (Pre) and their first post-test (Post 1) was significantly greater than zero ($r = .5844$, $t = 2.495^*$). The writing group's pre-test (Pre) and their second post test (Post 2) were also significantly correlated ($r = .7017$, $t = 3.412^*$). The critical t value for both of these analyses was 2.160. No other significant correlations were found within the groups. Although the magnitude of change in the comparison group's pre- and post-tests (Post 2) might suggest a significant relationship, the correlated t-test ($r = .3307$, $t = 1.108$) indicated the relationship was not significantly different from zero. An independent t-test also indicated no significant difference between the group's post test (Post 2) means scores.

Schumacher and Nash (1991) theorize reorganization occurs through the writing process. Given the lack of random assignment and non-experimental study design, inferential interpretation of theseresults will be cautious. However the statistical evidence seems to point toward tenuous support of this theory.

Discussion

Because this study co-existed with a teacher's planned instruction, in an existing classroom, the results will not be generalized and inferences are offered with a disclaimer. However, the study does describe a writing experience in one classroom for which the changes in knowledge appear to be promising. Explanation for changes in knowledge were expected to result, at least partially, from students' responses to the carefully constructed writing prompts initially based on informal and personal fraction knowledge. Some of the results may be due to the effect of nothing more than the fact that the students underwent extra experiences with fractions. However, the theory on which the study is based would suggest otherwise.

A relationship between informal fraction knowledge and formal fraction knowledge is promoted by many researchers (e.g. Carraher, 1996; Kieren, 1988; Mack, 1990, 1995; Streefland, 1978, 1982). They agree that first experiences with fractional activities should be based on students' personal and informal knowledge. Early writing prompts were designed accordingly. By its very nature, Connolly (1989) claims *write-to-learn* activity is informal. The changes in fraction knowledge found in this study may have been drastically different (or absent) without the specificity provided in those early writing prompts. In addition, the study was designed around the concept of fractions in order to limit interference from the mathematics of the children's active curriculum (subtraction of whole numbers).

The prompts were also intended to facilitate fraction usage in the children's language. Multi-step directions and the teacher's urging for students to revisit their narratives for fractional usage (writing to communicate as described by Durst & Newel, 1989; and Applebee, 1984) delineated early prompts. As the prompts moved beyond lower level fraction skills (Pothier & Sawada, 1983), and the students appeared to become more comfortable with describing fractional situations, the write-to-learn theory (Durst & Newel, 1989; and Applebee, 1984) would suggest students would learn about mathematics (in this case, fractions) through completion of the writing tasks. The two significant linear relationships within the writing group seems to support their suggestion.

Because acquiring knowledge of fraction concepts requires reorganization of prior knowledge, learning does not happen immediately (Carraher, 1996). The writing component of this study extended over a four week period, during which time no direct instruction about fractions was delivered. Given this design and the theoretical bases, the results of this study indicate the existence of a positive linear relationship between the pre and post tests,

marking a change in formal fraction knowledge, which may be attributable to the mental reorganization that occurred as a result of the writing experience. These results provide some evidence for the *Construction of Multiple Representations* theory (Schumacher & Nash, 1991) that writing compels students to reorganize their knowledge in such a way that manifests itself in a change in knowledge.

The writing group's substantial change in overall test scores (Pre to Post 1) might also be due to some combination of writing and regular instruction. These students had an opportunity to first experience fractional information through a variety of individual writing experiences. Later, they received regular classroom, whole-group instruction. Perhaps the writing activity facilitated some mental reorganization (Schumacher & Nash, 1991) making regular instruction accessible in a way not available to the comparison group.

Each of the writing situation prompts included physical models on which the students might base their writing. Streefland (1978) emphasized "the child's need for a framework existing of available models to solve problems concerning fractions" (p. 70). The models in this study were both real-world (e.g., water bottle and candy bar) and synthetic (e.g., pattern blocks and fraction circles). However, Kamii and Clark (1995) found that providing students with very structured models, already partitioned, seemed to rob students of the opportunity to construct their own knowledge. To promote student understanding of equivalent fractions, they suggest providing real world situations/problems for which students must struggle to find a resolution. The prompts used in this study were originally designed with a caution and empathy emphasizing a positive and productive experience in **writing** in mathematics. In retrospect, the situations may needed to come to resemble open-ended problems, requiring students to design their own models. "If children think about how to represent parts of a number of pizzas, this thinking will further their reasoning much more than ready-made pictures and fraction circles" (Kamii & Clark, 1995, p. 376). But only one of the higher level prompts required students to make their own models (Day 7 - rectangles with denominators of 12) and that prompt did not require them to design their own models. But, this study explored the sole impact of writing on mathematics learning not the impact of manipulating physical models on mathematics learning. However, some of the results may be explained, in part, by the physical models selected and the use of those physical models.

Implications for teaching

It is somewhat perplexing that neither of the groups' post test scores (Post 2) were significantly correlated to the score immediately preceding instruction. Although the writing group's scores from the pre (Pre) test taken in October and the post test (Post 2) following instruction in April were significantly correlated, no significant relationship was detected between Post 1 and Post 2. Similarly, the comparison group's post test (Post 2) scores, after receiving regular classroom instruction, were not significantly related to the pretest. Many and varied experiences are necessary to construct strong fractional knowledge (Mack, 1990; Streefland, 1978). Both teachers in this study approached the unit over fractions with a fairly traditional, textbook-oriented approach. Perhaps, a teacher who might choose to provide several tiers of writing prompts together with classroom instruction, as Evans (1984) did with geometry content, would find significant results. Moreover, if classroom instruction and writing activity were both designed around open-ended situations such as those discussed by Kamii and Clark (1995), students' grasp of fractions might be further changed.

Suggestions for future study

Additional writing studies with children of other ages and mathematics other than fractions would shed further insights upon the theories in question and provide evidence for the reasonableness of making generalizations beyond fourth graders and beyond basic fraction knowledge. Writing activities designed to accompany students' constructions of physical models may also be related to this area of study.

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